Dr. Kamlesh Kumar
Asst. Prof. (Guest Faculty)
Dept. of mathematics
Maharaja College,
V.K.S.U, Ara

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Complex Analysis (Continue)

Usefull property of conjugation :- If $z = x+iy$ then $\bar{z} = x-iy$ and so, $z\bar{z} = (x+iy)(x-iy) = x^2+y^2$.

Note :- $z\bar{z}$ is real.

Example :- Write $\frac{3+4i}{1+2i}$ in the standard form $x+iy$.

Soln - By property of conjugation to clear the denominator:

$$\frac{3+4i}{1+2i} = \frac{3+4i}{1+2i} \times \frac{1-2i}{1-2i} = \frac{3-6i+4i-8i^2}{(1)^2-(2i)^2} = \frac{3-2i+8}{1-4(-1)}$$

$$= \frac{11-2i}{1+4} = \frac{11-2i}{5} = \frac{11}{5} - \frac{2i}{5} = \frac{11}{5} + \left(\frac{-2}{5}\right)i$$ (standard form)

Real part $= \frac{11}{5}$, Imaginary part $= -\frac{2}{5}i$

The Magnitude of the complex number $x+iy$ is defined as $|z| = \sqrt{x^2+y^2}$. The magnitude is also called the absolute value, norm or modulus.

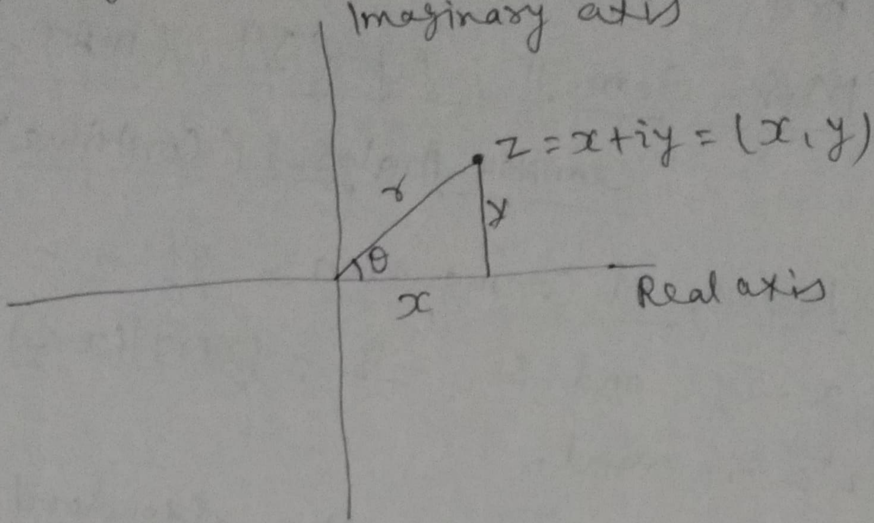
Ex. The norm of $3+5i = \sqrt{3^2+5^2} = \sqrt{9+25} = \sqrt{34}$.

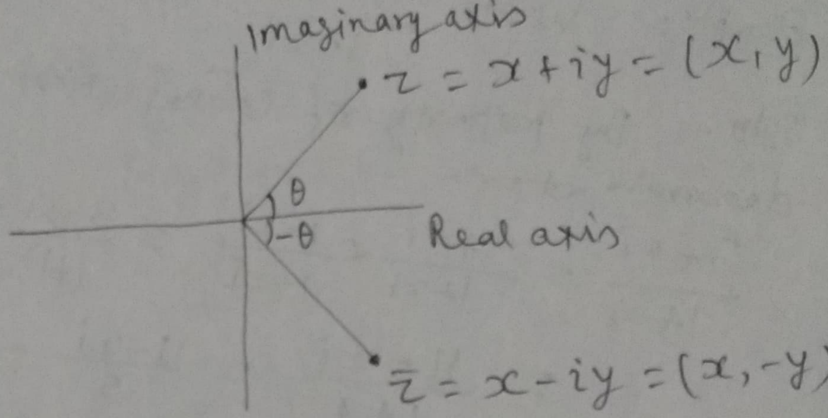
Important. The norm is the sum of x^2 and y^2. It does not include the i and is therefore always positive.

The complex plane : The geometry of complex number

Because it takes two numbers x and y to describe the complex number $z = x+iy$. We can visualize complex number as points in the xy-plane. When we do this we call it the complex plane.

Since x is the real part of z we call the x-axis the real axis. Likewise, the y-axis is the imaginary axis.

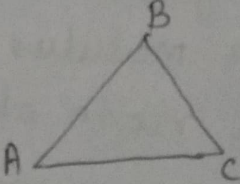




(complex plane)

<u>The Triangle inequality</u> :- The triangle inequality says that for a triangle the sum of the lengths of any two legs is greater than the length of the third leg.

Triangle inequality :→
$|AB| + |BC| > |AC|$



For complex number the triangle inequality translates to a statement about complex magnitude. Precisely: for complex numbers z_1, z_2, $|z_1| + |z_2| \geq |z_1 + z_2|$ with equality only if one of them is 0 or if $\arg(z_1) = \arg(z_2)$.

z_1 and z_2 are on the same ray from the origin.